DOCUMENT RESUME

ED 402 161 SE 059 185

AUTHOR Fast, Gerald R.
TITLE Analogies and Reconstruction of Mathematical Knowledge.
PUB DATE Oct 96
NOTE 18p.; Paper presented at the Annual Meeting of the Mid-Western Educational Research Association Conference (Chicago, IL, October, 1996).
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Concept Formation; Constructivism (Learning); Mathematical Concepts; *Mathematics Instruction; Problem Solving; Secondary Education; *Teaching Methods
IDENTIFIERS *Analogies

ABSTRACT

An investigation was conducted to determine the effectiveness of utilizing analogies to effect conceptual change in mathematics. Forty-one high school seniors participated in a knowledge reconstruction process regarding their beliefs about everyday probability situations such as sports events or lotteries. These mathematics students were given situations which were similar to those shown in previous research as revealing alternative mathematical conceptions. They were also given analogous researcher-generated anchoring situations which had been pre-tested and found to elicit acceptable responses. The conflicting responses motivated students to reconstruct their knowledge. The results of the investigation showed that analogies can be effective in producing a desired conceptual change in high school students' probability concepts. (Author/AIM)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 402 161

Analogies and Reconstruction of Mathematical Knowledge

Abstract

An investigation was conducted to determine the effectiveness of utilizing analogies to effect conceptual change in mathematics. Forty-one senior high school mathematics students were engaged in a knowledge reconstruction process regarding their beliefs about common everyday probability situations such as sports events or lotteries. The students were given situations which were similar to those shown in previous research as revealing alternative mathematical conceptions. They were also given analogous researcher-generated anchoring situations which had been pre-tested and found to elicit mathematically acceptable responses. The cognitive dissonance produced by the conflicting responses motivated students to reconstruct their knowledge. The results of the investigation showed that analogies can be effective in producing a desired conceptual change in high school students' probability concepts.

Presented at the
Mid-Western Educational Research Association Conference
Chicago, Illinois
October, 1996

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

G.R. Fast

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Gerald R. Fast, Ph.D.
College of Education and Human Services
University of Wisconsin Oshkosh
Oshkosh, Wisconsin 54901
(414) 424-7253 / 4444
fax: (414) 424-0858
fast@vaxa.cis.uwosh.edu

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

S605185

BEST COPY AVAILABLE

Introduction and Theoretical Background

Constructivist learning theory tells us that students really only come to understand when they are actively engaged in constructing their own knowledge (Brown, 1989; Glaser, 1991). This construction occurs through mental reflections which attempt to structure and organize knowledge into connected ideas and concepts. The mental reflections can be encouraged through various means and often at the elementary level they are activated through engagement with activities involving concrete manipulatives. Although many would agree that real learning and understanding only occurs when the student engages in this mental activity of sense-making of experiences, it is not always so clear how such a process of knowledge construction can be activated especially in secondary mathematics.

In constructivist learning theory the teacher's role is seen as a facilitator in the student's construction of knowledge (Barnard, 1989). The teacher does not tell students the answers but assists them in coming to their own conclusions, perhaps through collaboration with their peers. The teacher is seen as providing the scaffolding so that the students have the necessary support for continuing in their own building efforts. The role of the teacher is in knowing what supports to give and when to give them.

The research on which this presentation is based, focussed on the utilization of analogies in facilitating high school students' knowledge construction in the area of probability in mathematics. It might better be termed a knowledge reconstruction process since most students possess preconceptions about everyday occurrences which involve probability. For example, it is commonly believed that if a particular event has not occurred for some time then its probability of occurrence increases. For instance, when a coin toss has not resulted in heads 4 or 5 times, then it is expected that heads is more likely on the next toss. There are numerous everyday situations where this kind of reasoning is evident such as when a baseball player has not displayed his usual prowess for hitting a home run, it is believed he is about due to hit one. We also speak about this in regard to the weather. When we have a month of particularly mild weather in winter, some people say that we will have to pay for it with a period of weather colder than normal. All of these are examples of what Kahneman and Tversky (1972) have referred to as utilizing the representativeness heuristic to determine the probability of the occurrence of an event. Using this heuristic results in the mistaken assumption that a small sample must be representative of the population from which it is taken.

The representativeness heuristic manifests itself in various other ways as well such as believing that in a given family the birth order BGGBG is more common than BBBBB since BGGBG is more representative of the type of sample one normally observes which contains a mixture of boys and girls as compared to a family with children of only one gender. In this example, the student may also be utilizing the availability heuristic. This occurs if in attempting to answer the question, the student recalls various examples of the given situation which are available to him/her, and it is likely that most of these examples will be of families with children of both genders.
Research shows that alternative preconceived notions about probability are common and extremely difficult to alter or eradicate (Schrage, 1983). Various methods have been attempted including direct instruction and the use of computers to quickly generate large samples intended to demonstrate various ideas such as the Law of Large Numbers. Most of these methods have in themselves not been very effective (Konold, 1991).

Clement, in his research in teaching physics, showed that such a knowledge reconstruction process could be effectively initiated through the use of analogies (Clement, 1987; Clement, et al., 1989). He describes a tutorial which uses analogies to bridge the gap between anchoring examples which the students understand as illustrating a certain law or principle and the target examples which the students do not perceive as showing the same idea. Bridging examples which have some of the characteristics of the anchoring example and some of the characteristics of the target example are used as supports in conceptually bringing the two analogous examples closer together.

To illustrate, consider the common misconception, contrary to Newton's Laws, that a table does not exert a force on a book lying on it even though it is believed the book exerts a force on the table. The anchoring example which would likely activate the correct schema would be pushing down on a spring with your hand. You are exerting a force downward on the spring but you can also feel the spring exerting a force upward on your hand. Clement (1987) found that "In almost all cases students believe in the anchoring example that a spring can push back on one's hand, but many are still unconvinced that there is a valid analogy relation to the case of the book on the table" (p. 87). This is because the anchoring example is too far removed from the target example, the book resting on the table, and so intermediate examples which serve as bridges must be given. One might show the book on the spring after showing the 'hand-on-the-spring' example and then the book resting on a piece of foam. Now show the book resting on a flexible plank. In each case we are gradually moving away from the 'hand-on-the-spring' situation towards the 'book-on-the-table' situation. If these examples are not sufficient, the tutorial may also introduce a microscopic model of rigid objects as being composed of atoms connected by spring-like bonds.

The Research Method

The Purpose

An approach similar to the one used by Clement (see above) was attempted to determine whether knowledge reconstruction could be effected through the use of analogous anchoring situations in the area of probability.

The Participants

The *WDYTTCA* instrument (see Appendix A) was given to a group of 41 senior students of both genders from a large suburban high school. All the students who were present on the day the *WDYTTCA* instrument was administered agreed to participate. Seventeen of these students were interviewed. All participation was completely on a voluntary basis.

The Procedure

The participants began by first completing *Version A* of the *WDYTTCA* instrument. When they were finished they immediately went on to *Version B*. They were instructed not to go back to *Version A* after beginning *Version B*. The objective was to determine the effectiveness of the situations in *Version B* of the instrument in eliciting mathematically-correct responses and appropriate justifications especially in regard to those questions whose analogous counterparts in *Version A* of the instrument had received incorrect responses and misconception-revealing justifications. No time limit was imposed on completing the instrument.

The Instrument

A variety of situations identical or similar to the ones extensively quoted in the literature as misconception-prone probability situations were matched with researcher-generated anchoring situations conceptually isomorphic to the misconception-prone situations. It was hypothesized that for various reasons the anchoring situations were more likely to activate mathematically-correct schemata than the misconception-prone situations. These reasons were based on various techniques used in generating the anchoring situations such as presenting the problem from a different perspective; utilizing concrete or familiar situations; changing the numerical quantities in order to present an extreme case, etc. Finally, ten misconception-prone situations were compiled into *Version A* of the *WDYTTCA* (*What Do You Think The Chances Are?*) instrument. Their ten analogous counterparts generated by the researcher were placed into *Version B* of the *WDYTTCA* instrument (see Appendix A). Questions regarding the likelihoods of events were posed in a multiple-choice format. Justifications for the responses were in constructed-response format.

Confidence lines were included so that the respondent could indicate his/her degree of confidence in the responses on a continuum from 0 to 3 with demarcations 0 (just a guess); 1 (not very confident); 2 (fairly confident); and 3 (I'm sure I'm right). An example of one of the target situations and its analogous anchoring counterpart is given below. Explanations regarding the status of the question as a target or anchor are also given.

Target

Your sports team finishes first in its league at the end of the season and so you consider it is the best team. However you must compete in a playoff series against the second place team in the league to determine the champion. Would a 5 game series or a 9 game series give you a better chance of winning the championship, or doesn't it make any difference?

a) a 5 game series gives you a better chance;
b) a 9 game series gives you a better chance;
c) it makes no difference.

Those who rely on the representativeness heuristic believe that both the larger sample 9 game series and the smaller sample 5 game series will reflect the population (i.e. that your team is better) equally well in all cases. They will probably respond that it makes no difference, not taking into consideration that the larger sample, the 9 game series, would be more likely to show which team is actually better.

Anchor

Your sports team finishes first in its league at the end of the season and so you consider it is the best team. However you must compete in a playoff series against the second place team in the league to determine the champion. Would a sudden-death 1 game playoff or a 5 game series give you a better chance of winning the championship, or doesn't it make any difference?

a) a sudden-death 1 game playoff gives you a better chance;
b) a 5 game series gives you a better chance;
c) it makes no difference.

Experience tells you that you do not win every game even if you are the best team and therefore a sudden-death 1 game playoff is very risky whereas a 5 game series gives you a better chance to show your talent. It is the extremity of the numerical quantity "1" in the 1 game series which makes the correct choice more obvious. Consequently, this situation provides an anchor for the

statistically-correct concept that a larger sample is more likely to reflect the characteristics of the population from which it is taken than a smaller sample.

Interviews

The purpose of the interview was firstly to establish whether or not the possible misconceptions which had been revealed in the *WDYTTCA* instrument were true misconceptions. Secondly, and most importantly, the interview attempted to determine the effectiveness of using analogies to effect a conceptual change where probability misconceptions had been revealed. The participants were presented with the situations in *Version A* of *WDYTTCA* to which they had responded incorrectly (possible misconceptions) when at the same time they had responded correctly and with confidence (anchors) to the analogous counterparts in *Version B* . They were engaged in a process of analogical reasoning by the interviewer in an attempt to guide the knowledge reconstruction process which hopefully would ultimately result in the participant changing incorrect responses to correct responses in the *Version A* situations (see Appendix B for Interview Protocol). This change from an incorrect to a correct response combined with an indication of being at least "fairly confident" in the correct response was deemed as evidence of successful conceptual change necessary in overcoming the misconception.

Findings

Correct Responses on Version A and B

Of the 410 responses (41 students x 10 questions) in *Version A* of *WDYTTCA* , 178 were mathematically correct leaving 232 as possible misconceptions. *Version B* of *WDYTTCA* resulted in 330 correct responses out of a possible 410 responses. Each question in *Version B* also resulted in more correct responses than its analogous counterpart in *Version A* . Each student also obtained at least as many or more correct responses in *Version B* as in *Version A* .

Regarding the number of possible misconceptions revealed in *Version A*, the results appear to confirm what previous research has shown, that is, probability misconceptions are common amongst high school students, although at this point it is not clear how many of the possible misconceptions are true misconceptions. Incorrect responses may have been caused by careless reading or a misunderstanding or lack of understanding of the situation.

Version B received almost twice as many correct responses as *Version A*. On a question-by-question and a student-by-student comparison, *Version B* also consistently outscored *Version A* in correct responses. This is a strong indication that the situations in *Version B* are serving their intended purpose of activating mathematically-correct schema.

Potential Anchors

Of greatest interest however were those situations in which an incorrect response was given in *Version A* indicating a possible misconception, and a correct response was given in the analogous question in *Version B* indicating a potential anchor. Of the 232 possible misconceptions in *Version A*, 166 were matched with potential anchors in *Version B* .

These results provide further evidence that the questions in *Version B* are serving their intended purpose since over half of the questions which were answered incorrectly in *Version A* were answered correctly in analogous form in *Version B*.

Anchors

When a student indicated that he/she was at least fairly confident (a rating of 2 or greater on the Confidence Line) in a situation deemed as a potential anchor, then this situation was referred to as a true anchor or simply an anchor for that individual. Using this definition, 113 of the 166 potential anchors could be referred to as anchors. Thus the overall Instrument Anchoring Rate (anchors / possible misconceptions) was 0.49.

Interviews

Seventeen of the 41 high school students volunteered to be interviewed. Each volunteer was interviewed regarding those situations where anchors were produced in *Version B* for the possible misconceptions which had been revealed in *Version A* . The total number of situations dealt with was 46 ranging from 1 to 5 per student. Interviews ranged in time from approximately 15 minutes to one hour. The main purpose of the interview was to determine whether the anchors in *Version B* would be effective in producing conceptual change in the misconceptions revealed in *Version A*. The results were categorized as Successful, Partially Successful, or Unsuccessful (see Appendix C for definitions). For analysis purposes, these categories were assigned scores of 1.0, 0.5, and 0.0 respectively. Of the 46 situations attempted, 24 were categorized as Successful, 18 as Partially Successful, and 4 as Unsuccessful. This resulted in an overall Interview Success Rate of 0.72.

The majority of students engaged in the interview process found the analogies useful in reconstructing their thinking regarding the troublesome probability situations which tend to produce responses at variance with accepted statistical principals and laws. There was definite evidence of conceptual change as students indicated that they would like to change their responses - usually in the target situations. This occurred on two levels. In a few cases, students immediately changed their responses in the target situations after being given the anchoring analogy. Most cases however required a series of questions from the researcher to facilitate the knowledge reconstruction process. Sometimes alternative representations in addition to those in the instrument were required, including ones which originated with individual students.

Reasons for Analogy Effectiveness

There are various reasons why probability situations presented in the usual form may result in responses which are at variance with accepted mathematical ideas. These reasons can be utilized in designing alternative, analogous forms of the situations which are effective for reconstructing students' ideas.

Certain cues in a given situation are often the cause for the activation of misconception-prone schemata. For example, in *Version A*, Question 6 (see Appendix A) includes a description of the environment in which the vehicle is operated. Because this environment includes conditions which would be conducive to 'driving a 4-w-d truck', this is often the statement which is chosen as having a greater chance of being true rather than the statement which says that Tom is 'driving a truck'. When these conditions or 'cues' are removed from the situation, the student will often choose the mathematically-correct answer when he/she did not do so when the cues were present thus producing an anchor in the 'cues removed' condition. The analogous situation in *Version B* did not have such cues and it consequently served quite well as an anchor. Question 9 fell into the same category of using a 'cues removed' situation in *Version B* and again this was effective in producing anchors.

A simpler case, or one in which the cues which activate the misconception-prone schema are not as powerful, could serve as an anchor. Questions 1 and 4 are examples of this. Simpler cases presented to some students during the interviews were also effective in reconstructing knowledge in Question 5.

The analogy in Question 5 (*Version B)* was very effective for overcoming misconceptions evident in its counterpart in *Version A*. This is an example of a situation being represented from a different perspective which immediately makes the correct answer obvious to some students.

The choice of numbers in a situation, particularly the use of extreme cases, was also effective as demonstrated in Question 3. The choice of a 1 or 5 game series as compared to the choice between a 5 or 9 game series assisted some students in choosing the longer series as being advantageous for the better team because the 1 game sudden-death playoff was seen as being too risky. Question 8 also fell into this category in regard to changing the time between the two sets of 'at-bats' from a few minutes in *Version A* to two months in *Version B*.

In Question 7, the analogy in *Version B* contains only part of the problem given in *Version A*. *Version B* asks for an estimate of the number on the third ball whereas *Version A* asks for an estimate of the mean of the three balls. The former is a necessary step in answering the latter. The *Version B* question was also presented in a 'cues removed' condition. This combination proved to be quite effective in producing a relatively high anchoring rate.

The utilization of graphics was attempted in Question 10 as a means of generating anchors but this was not as effective as had been anticipated. The low anchoring success rate resulted primarily from a lack of confidence in those students for whom it was a potential anchor. The situation represented graphically in *Version B* served as a potential anchor for 17 of the 24 students who had a possible misconception in *Version A* but this number dropped to 8 for those who were at least 'fairly confident' in their mathematically-correct response.

Finally, a more familiar situation can also serve as an anchor for a less familiar situation. Question 2 is an example of this. A situation involving a particular order of digits was an effective anchor for a situation involving a particular birth order.

Conclusion

The research set out to explore the feasibility of using a novel approach to reconstructing high school students' ideas about probability which are at variance with accepted mathematical ideas. This approach involved firstly the generation of anchoring situations which are analogous to

misconception-prone probability situations. Secondly, the anchoring situations were utilized in individual interviews in attempts to reconstruct ideas which were indicative of misconceptions.

The results of the study showed that the use of analogies is effective for reconstructing mathematical knowledge with senior high school students regarding their probability misconceptions. The overall (interview) success rate was 0.72 . These results are quite remarkable in view of the fact that misconceptions are generally highly resistant to change. The approach used in this study does not deny the value of other approaches; the utilization of analogies combined with another approach may actually prove more effective than the utilization of either approach on its own. There was some evidence of this in the interviews. Nevertheless, this study has revealed that the use of analogies can make a valuable contribution to overcoming probability misconceptions. The ultimate value of this contribution will hopefully be determined by further research. It would also be of interest to investigate analogy effectiveness in reconstructing knowledge in other areas of mathematics.

References

Barnard, J. J. (1989). Poor Concept Formation in Mathematics: A Diagnostic Perspective. Pretoria, SA: Human Sciences Research Council.

Brown, D.E. & Clement, J. (1989). Overcoming Misconceptions Via Analogical Reasoning: Abstract Transfer Versus Explanatory Model Construction. *Instructional Science, 18*(4), 237-261.

Brown, J.S., et al. (1989). Situated Cognition and the Culture of Learning. *Educational Researcher, Jan-Feb,* 32-42.

Clement, J. (1987). The Use of Analogies and Anchoring Intuitions to Remediate Misconceptions in Mechanics. Paper presented at the Annual Meeting of the American Educational Research Association, Washington, D.C.

Clement, J., et al. (1989). Not All Preconceptions Are Misconceptions: Finding "Anchoring Conceptions" for Grounding Instruction on Students' Intuitions. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA.

Cox, C. & Mouw, J.T. (1992). Disruption of the Representativeness Heuristic: Can We Be Perturbed Into Using Correct Probabilistic Reasoning? *Educational Studies in Mathematics, 23*(2), 163-178.

Glaser, R. (1991). The Maturing of the Relationship Between the Science of Learning and Cognition and Educational Practice. *Learning and Instruction, 1*(2), 129-144.

Kahneman, D. & Tversky, A. (1972). Subjective Probability: A Judgment of Representativeness. *Cognitive Psychology, 3,* 430-454.

Konold, C., et al. (1991). The Origins of Inconsistencies in Probabilistic Reasoning of Novices. In D. Vere-Jones (Ed.), *Proceedings of the Third International Conference on Teaching Statistics* (Vol. 1, pp. 357-362). Voorburg, NL: International Statistical Institute.

Schrage, G. (1983). (Mis-) Interpretation of Stochastic Models. In R.W. Scholz (Ed.), *Decision Making Under Uncertainty.* Amsterdam, NL: Elsevier Science Publishers B.V.

Tversky, A. & Kahneman, D. (1971). Belief in the Law of Small Numbers. *Psychological Bulletin, 76*(2), 105-110.

Tversky, A., & Kahneman, D. (1973). Availability: A Heuristic for Judging Frequency and Probability. *Cognitive Psychology 5,* 207-232.

Tversky, A. & Kahneman, D. (1977). Judgment Under Uncertainty: Heuristics and Biases. In P.N. Johnson-Laird & P.C. Watson (Eds.), *Thinking: Readings in Cognitive Science* (pp. 326-337). London: Cambridge University Press.

Appendix A: The *WDYTTCA* Instrument

WDYTTCA Version A

1. A fair coin is tossed 5 times and the result is HHHHH . On the next toss, which outcome, if either, do you think has a better chance of occurring, H or T ?
   H = Heads T = Tails

   a) H has a better chance of occurring;
   b) T has a better chance of occurring;
   c) They both have the same chance of occurring.

   Why?_____
   _____
   _____

   I_____I_____I_____I
   just a guess not very confident fairly confident I'm sure I'm right

* To save space, the lines for indicating the justification and the confidences have been deleted in the remaining questions.

2. In families with five children, which birth order, if either, occurs more often: BGGBG or BBBBB ? B = Boy and G = Girl .

   a) BGGBG occurs more often;
   b) BBBBB occurs more often;
   c) They occur equally often.

3. Your sports team finishes first in its league at the end of the season and so you consider it is the best team. However you must compete in a playoff series against the second place team in the league to determine the champion. Would a 5 game series or a 9 game series give you a better chance of winning the championship, or doesn't it make any difference?

   a) a 5 game series gives you a better chance;
   b) a 9 game series gives you a better chance;
   c) it makes no difference.

4. In the 6/49 lottery, Tom has chosen the numbers: 1 2 3 4 5 6 , and Bill has chosen the numbers 3 8 19 27 32 41 . To win the jackpot you must match all the six numbers you have chosen with the six numbers randomly selected by a machine from the numbers 1 to 49 inclusive. The order of the numbers is not important. How would you compare their chances of winning the jackpot?

   a) Tom has a better chance;
   b) Bill has a better chance;
   c) They both have exactly the same chance.

5. Suppose that a new lottery is created in which you have to correctly choose 43 numbers out of 49. How would the difficulty of this compare to correctly choosing 6 numbers out of 49 as is the case in the present 6/49 lottery? The order of the numbers is not important.

a) correctly choosing 6 numbers out of 49 is more difficult;
b) correctly choosing 43 numbers out of 49 is more difficult;
c) they are equally difficult.

6. Tom lives in the Yukon. He drives long distances, often on snow-covered roads and rough or muddy trails. Sometimes he must cross shallow rivers and climb steep hills with his vehicle. Which of the two statements below has a greater chance of being true?

a) Tom drives a truck;
b) Tom drives a 4-wheel-drive truck.

7. A large container is filled with numbered balls. The average (mean) of all the numbers on the balls is 100. Three balls are drawn with replacement from the container. If the numbers on first two balls drawn are both 130, what is your best estimate for the average (mean) of the numbers on the three balls drawn?

a) 100 b) 115 c) 120 d) 130

8. Suppose John Olerud has a batting average of .333 (1 hit in 3 times at bat) against Jimmy Key. In a certain game he comes up to bat 6 times against Key. He has no hits the first 3 times at bat. What is your best guess as to how many hits he will get in his last 3 times at bat?

a) 0 b) 1 c) 2 d) 3

9. Jack is tall and muscular. He exercises frequently and prides himself on his strength and his running and throwing abilities. He is often seen at certain sports events and driving his fast sports car. Which of the two statements below has a greater chance of being true ?

a) Jack is a farmer;
b) Jack is a professional football player.

10. If 50 people are selected at random, which do you think is more likely: that no two people in the group of 50 would have been born on the same day of the year, or that there would be at least two people in the group of 50 who were born on the same day of the year?

a) it is more likely that no two people were born on the same day of the year;
b) it is more likely that there would be at least two people who were born on the same day of the year.

WDYTTCA Version B

1. A fair coin is tossed once and the result is H . On the next toss, which outcome, if either, do you think has a better chance of occurring, H (heads) or T (tails) ?

   a) H has a better chance of occurring;
   b) T has a better chance of occurring.
   c) They both have the same chance of occurring.

2. In a lottery called Pick 4, a 4-digit number like 2798 is generated. To win, the participant must have chosen the same 4-digit number. Albert has chosen the number 2222 and Bill has chosen the number 2332. Compare their chances of winning.

   a) Albert has a better chance of winning;
   b) Bill has a better chance of winning;
   c) They both have the same chance of winning.

3. Your sports team finishes first in its league at the end of the season and so you consider it is the best team. However you must compete in a playoff series against the second place team in the league to determine the champion. Would a sudden-death 1 game playoff or a 5 game series give you a better chance of winning the championship, or doesn't it make any difference?

   a) a sudden-death 1 game playoff gives you a better chance;
   b) a 5 game series gives you a better chance;
   c) it makes no difference.

4. In an urn there are ten balls numbered from 1 to 10. How would you rate the difficulty of drawing, without replacement, the balls numbered 3, 1, and 2 , compared to drawing the balls numbered 2, 5, and 9 ? The order in which the balls are drawn is not important.

   a) 3, 1, and 2 is more difficult.
   b) 2, 5, and 9 is more difficult.
   c) They are equally difficult.

5. In order to win the 6/49 lottery jackpot, you have to shade in the 6 numbers which are drawn, out of the possible 49 numbers on a card. Suppose <u>they still draw 6 numbers</u> , but in order to win, you must now shade in the 43 numbers on the card which are not drawn. Would this make it any more difficult to win?

   a) it is more difficult to win by shading in the 6 numbers which are drawn.
   b) it is more difficult to win by shading in the 43 numbers which are not drawn.
   c) they are the equally difficult.

6. You are driving down the highway when you are passed by a car. Which of the two statements below has a greater chance of being true?

   a) The car is red;
   b) The car is a red convertible.

7. A large container is filled with numbered balls. The average (mean) of all the numbers on the balls is 100. Two balls are randomly drawn with replacement from the container and their numbers are recorded but not seen by you. If you now draw a third ball, what is your best estimate for the number on this ball?

   a) 40 b) 70 c) 100 d) 130 e) 160

8. Suppose Joe Carter has a batting average of .250 (1 hit in 4 times at bat) against Jimmy Key. In a game in June, Carter gets no hits out of 4 times at bat against Key. Two months later in August, Carter again comes to bat 4 times against Key. How many hits do you expect him to get this time?

   a) 0 b) 1 c) 2 d) 3 e) 4

9. If a person is chosen at random from the Canadian population, do you think it is <u>more likely</u> that the person is more than 100 years old or less than 100 years old?

   a) The person is more than 100 years old;
   b) The person is less than 100 years old.

10. On the following page you see part of a tiled sidewalk made up of small squares*. It begins to rain. After 50 raindrops have fallen on the squares, which do you think is more likely: that there is <u>no</u> small square on which 2 raindrops have fallen; <u>OR</u>, that there is at least <u>one</u> small square on which 2 raindrops have fallen?

    a) there is <u>no</u> small square on which 2 raindrops have fallen;
    b) there is at least <u>one</u> small square on which 2 raindrops have fallen.

    * The part of the tiled sidewalk shown consisted of 365 squares.

Appendix B: Sample Interview Protocol

Question 8

I: Let's have another look at one of the questions you answered previously on *WDYTTCA*. Here is Question 8 of *Version A*. Would you read the question again and look at the answer you gave. [Student reads the question.]

I: So you said that Olerud should get 2 hits in the last 3 times at bat to maintain his average of .333. Do you still agree with this answer? [If the student says "no", then the interviewer will ask why not and what the new answer is and why. This is to determine if the student has been influenced by *Version B* of *WDYTTCA* or any other factor since answering the question. If the student already knows the correct answer and can give an appropriate explanation, then it can be concluded that the student has already overcome the misconception. If the student says she agrees with her original answer, she will then be presented with the *Version B* situation.]

I: Now let's have a look at Question 8 of *Version B*. Would you please reread the question and the answer you gave. [Student reads the question and the answer she gave.] You said that Carter would get 1 hit against Key. Do you still agree with your answer? [If the student says "no" and gives the wrong answer with an explanation indicating a misconception as in *Version A*, then the situation has lost its role as an anchor. The student will then be asked why she changed her mind. If the student, however still agrees with her former correct response, she will then be asked .....]
I: Look at this Question 8 on *Version B* and this Question 8 on *Version A*. Do you see a similarity between the two questions? [If the student says no, similarities can be pointed out, but if the student says yes as expected .....]

I: Okay, what similarities do you see? [Similarities are established and then ....]

I: What about your answers for the two questions, are you using the same reasoning to answer both? [If she answers yes, further probing can reveal the difference. If she answers no .....]

I: So why did you use different reasoning to answer the two questions? [The difference between the two questions is simply a time difference between the batting sessions so it is expected that the student will point out this difference. Then .....]

I: Suppose Carter comes to bat 4 times against Key 1 month later instead of 2 months later, how many hits do you expect him to get? [The idea is to gradually decrease the time interval between the two batting sessions until they occur in the same game as in *Version A* while at the same time having the student maintain her anchoring response in *Version B*. If the anchor is not a brittle anchor then this can be done.]

I: So, how may hits do you think Olerud would get in the last 3 times at bat? [referring back to Question 8, *Version A*. If the student changes her answer to "1" and she gives an appropriate explanation as she did in *Version B*, she will then be asked how confident she is in her new answer. She will be presented with the confidence line. If she indicates at least a 2 (fairly confident) it will be concluded that she has at least temporarily overcome her misconception and that the anchoring situation can serve to remind her of the mathematically-correct concept in future situations of this type where independence is required in the absence of any causal information.]

Appendix C: Definitions

Successful Reconstruction - When a participant changes his/her incorrect response to a correct response in *WDYTTCA Version A* using the analogy in *Version B* and/or an alternate analogy provided by the interviewer and indicates that he/she is at least fairly confident (level 2 or higher on confidence scale) in the revised response.

Partially Successful Reconstruction - When a participant changes his/her incorrect response to a correct response in *WDYTTCA Version A* but indicates that he/she is not very confident (level 1 or lower on the confidence scale) in the revised response or it appears that the revised response is probably partly due to aspects other than an analogous situation.

Unsuccessful Reconstruction A - When a participant does not change his/her incorrect response to a correct response in *WDYTTCA Version A* .